

Motion in one dimension

Name:

PES 1150 Prelab Questions

Lab Station: 005

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1. What are the two important limitations to keep in mind when using the Motion Detectors?

The motion detectors cannot pick up accurately an object located within about 0.5 meters from the detector. That is, anything closer than 0.5 meters to the sensor will likely give a “bad” reading. Second, *all* objects are picked up by the sensor (including stray hair, clothing, arms, books, binders, cables, etc...). Likewise, the sensor picks up small variations in the object it is measuring (like curves of the body – which could effectively introduce a variance to the measurement).

2. What is the acceleration of gravity on the earth (include units)?

The acceleration of gravity on the Earth at zero latitude and sea level with a NADIR pointing reference frame (pointing **toward** the center of the earth to be the positive z-axis) is a given “constant”. This is namely given as:

$$\underline{g = 9.81 \frac{m}{s^2}}$$

Note that it is particularly important to call out that this theoretical value is only “constant” at zero latitude and sea level. This is caused by the Force due to gravity (for a two body system).

$$\underline{F_{Gravity} = -G \frac{Mm}{r^2}}$$

Where “G” is the Newtonian Gravitational Parameter, “M” is the mass of the Earth (or the larger of the two massive bodies), “m” is the mass of the object being acted upon by the Earth (or the smaller of the two massive bodies), and “r” is the relative distance between the “center of mass”-es for the two bodies. [Note that the negative is provided due to the fact that we are dealing with a magnitude of a Force Vector. By convention, we take “up” often called ZENITH (the direction **away** from the center of mass of the earth) to be the default direction of a positive z-axis (notice that this is different than

above). Since gravity tends to create a force in the direction of the center of mass of the earth, the direction of the force is thus in the negative z direction (in a ZENITH coordinate frame). If we wanted to be completely explicit, we would make the forces into vectors and specifically call out the coordinate frame and direction of the forces, in addition to their magnitudes – because we all know that vectors must have both: a) a magnitude and b) a direction.]

$$\vec{F}_{Gravity} = -G \frac{Mm}{r^2} \hat{z}$$

(In the standard SEZ [South-East-Zenith] Cartesian Coordinate Frame on Earth)

How to calculate “g” on the Earth (or any other planet for that matter):

By Newton’s 2nd law:

$$F = ma$$

Thus,

$$F_{Gravity} = ma_{Gravity} = -G \frac{Mm}{r^2}$$

$$a_{Gravity} = -g = -G \frac{M}{r^2}$$

Hence:

$$g = G \frac{M}{r^2}$$

<u>Gravitational Constant Parameter</u>	<u>Mass of the Earth</u>	<u>Radius of the Earth (at the equator at sea level)</u>
<u>$6.6726 \times 10^{-11} \text{ [m}^3/\text{kg s}^2\text{]}$</u>	<u>$5.9737 \times 10^{24} \text{ [kg]}$</u>	<u>6374333 [m]</u>

$$g = \left(6.6726 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} \right) \frac{(5.9737 \times 10^{24} \text{ kg})}{(6374333 \text{ m})^2} = 9.81 \frac{\text{m}}{\text{s}^2}$$

$$\boxed{g = 9.81 \frac{\text{m}}{\text{s}^2}} \quad \text{and} \quad \boxed{a_{Gravity} = -g = -9.81 \frac{\text{m}}{\text{s}^2}}$$

** NOTE: If we increase “r” (i.e., increase our altitude from the “sea level” radius of the Earth [e.g., climb up a mountain]) we decrease the value of “g”. Likewise, we have assumed that the Earth is spherical for all these calculations (in which case the calculation

of “g” would be independent of latitude.) However, from very careful measurements, it is shown that the Earth is actually a “squished-sphere” and the average distance to the poles is less than the average distance to the equator [this is called the “oblate Earth”]. An example of the “oblate Earth” model is shown in the following figure.



Note that the polar radius is slightly less than the equatorial radius of the Earth. This means that as we increase or decrease latitude, the distance from the center of mass of the Earth to the center of mass of an object on the Earth will likewise increase or decrease. This in turn will change the value of “g”. This variance is beyond the scope of PES 1150, but can be further investigated in Astrophysics courses or Aerospace courses. If you were curious, the “eccentricity” [squishiness] of the Earth is: $e_{\text{Earth's Shape}} = 0.08182$. (This means it is almost perfectly spherical – but not quite).

Over multiple measurements, I have personally found that a value of about 9.79 m/s^2 is a pretty accurate acceleration due to gravity in Colorado Springs.

3. Find the average value of the following numbers:

10.02, 9.98, 9.91, 10.1, 9.87.

The average is calculated using the following equation:

$$\langle x \rangle = \frac{1}{N} \sum_{i=1}^N x_i$$

Where x is the data we are finding the average of, and x_i are the individual components of the given data. Furthermore, N is the total number of elements in the given data. For our case, N is 5, and x is Acceleration; hence:

$$\langle a \rangle = \frac{1}{5} \sum_{i=1}^5 a_i$$

$$\langle a \rangle = \frac{1}{5} \left(10.02 \frac{m}{s^2} + 9.98 \frac{m}{s^2} + 9.91 \frac{m}{s^2} + 10.1 \frac{m}{s^2} + 9.87 \frac{m}{s^2} \right)$$

$$\boxed{\langle a \rangle = 9.976 \frac{m}{s^2}}$$

For the sake of education, compare this average to the value of g. What type of error is present in the data? Random error, systematic. Explain your answer.

We will take the standard accepted value of the acceleration due to gravity at sea-level and at the equator to be the theoretical value for g. Likewise, we will take the measured value to be the average calculated in the question above. Hence:

$$\underline{g_{Theory} = 9.81 \frac{m}{s^2}} \quad \underline{g_{Measured} = 9.976 \frac{m}{s^2}}$$

Using the percent difference, we can compare the “measured” acceleration due to gravity with the “theoretical” acceleration due to gravity.

$$\% \text{ difference} = \frac{|g_{Theory}| - |g_{Measured}|}{|g_{Theory}|} \times 100\% = \frac{\left(9.81 \frac{m}{s^2} \right) - \left(9.976 \frac{m}{s^2} \right)}{\left(9.81 \frac{m}{s^2} \right)} \times 100\%$$

$$\boxed{\% \text{ difference} = -1.692\%}$$

If we consider the data provided in the question above, we can see that the data is exhibiting the characteristics of both systematic and random error. Notice that the data appears to be “high” for every measured value. That is – there are not approximately the same number of measured values higher than 9.81 m/s^2 as there are number of measured values lower than 9.81 m/s^2 . All the values appear to be “shifted-off” to the high end by a given amount. This is systematic error. Furthermore, not all the values are within a small delta range. They range from 9.87 m/s^2 to 10.1 m/s^2 . This is random error. We took the average of many values – this will compensate for the random error, because the more data points available, the more likely the average of the values will converge to unity. The systematic error will always be present, unless we can assuredly calibrate our measurement tool to a well known source.

By examining our result of the percent difference calculation, we can see that we are not far off from the theoretical value of the acceleration due to gravity. The negative means that we were slightly higher (by exactly 1.5%) the theoretical value. This could be explained by Colorado Springs’ latitude being about 40° North and approximately an altitude of 1.873 km (6150 ft) above sea level. The actual geodetic location of Colorado

Springs Municipal is given in the following table. (This is available on the Colorado Springs Web Page at <http://www.cospgs.com/weather.html>).

<u>LATITUDE</u>	<u>LONGITUDE</u>	<u>ELEVATION (ft.)</u>
<u>38° 48' 43" N</u>	<u>104° 42' 40" W</u>	<u>6145</u>

** Aside **

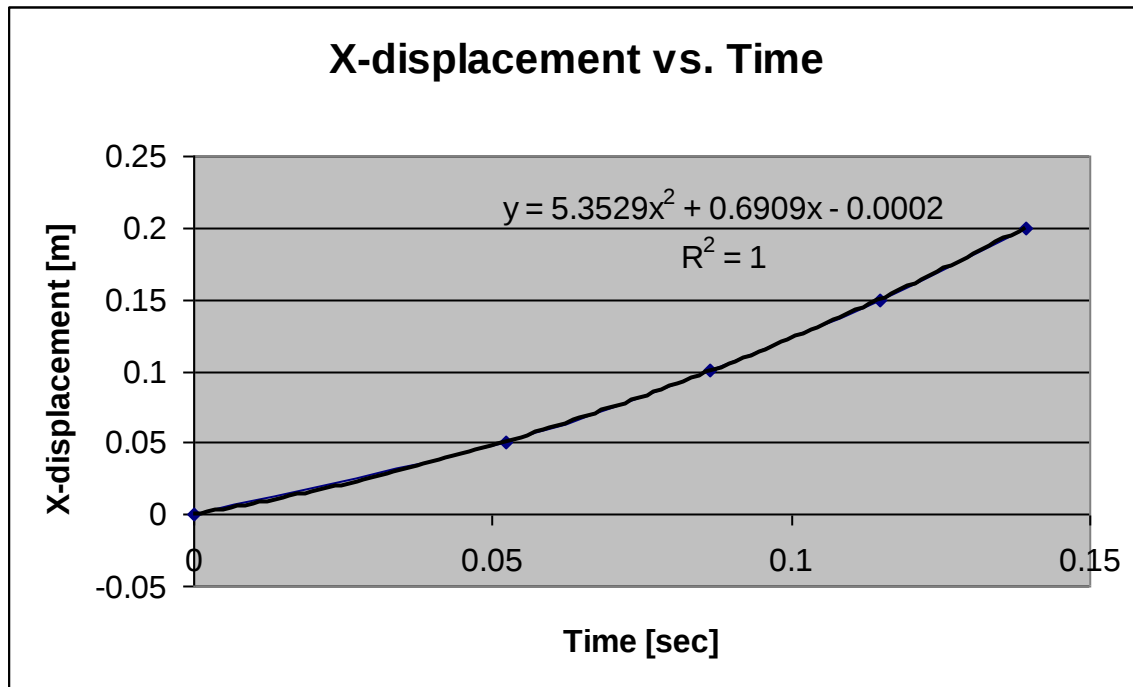
Recently, scientists believe that there may be proof for “underground gravitational anomalies”. These massive bodies suspended in the magma, and in addition the fact that the Earth’s core may be off-center, could lead to small, but detectable variances in the measure of the acceleration due to gravity at specific, “abnormal locales”. Who knows ... maybe one of these “underground gravitational anomalies” could exist between the Earth’s center of mass and the experimental masses of the Physics 1150 labs at the University of Colorado at Colorado Springs – causing systematic “erroneous” measurements. ☺

** End Aside **

4. Calculate a velocity vs. time from the following position vs. time as shown in the manual.

t (sec.)	x (meter)	Average Velocity (m/s)
0	0	
.05227	.0500	
.08638	.1000	
.1149	.1500	
.1394	.200	

First, let’s plot the distance versus time, to see what it looks like:



To calculate the velocity, we need both a distance and a time measurement. Since our object is accelerating, the velocity will be changing over the course of the measurements. To calculate the velocities, we need to take an average of two sequential times and consider the change of displacement over that time (remember that the slope of x-displacement versus time graph is a velocity). The first average velocity is given as:

For the first and second data points (to find the first average velocity):

$$\text{Average Velocity} = \langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o}$$

Time [sec]	X-displacement [m]
$t_o = \underline{0}$	$x_o = \underline{0.0000}$
$t_f = \underline{0.05227}$	$x_f = \underline{0.0500}$

$$\langle v \rangle_1 = \frac{(0.05 \text{ m}) - (0.0 \text{ m})}{(0.05227 \text{ sec}) - (0 \text{ sec})} = 0.95657 \frac{\text{m}}{\text{s}}$$

$$\boxed{\langle v \rangle_1 = 0.95657 \frac{\text{m}}{\text{s}}}$$

The average of the two sequential times is:

$$\text{Average Time} = \langle t \rangle = \frac{t_f + t_o}{2}$$

$$\langle t \rangle_1 = \frac{(0.05227 \text{ sec}) + (0 \text{ sec})}{2} = 0.026135 \text{ sec}$$

$$\boxed{\langle t \rangle_1 = 0.026135 \text{ sec}}$$

For the second and third data points (to find the second average velocity):

Time [sec]	X-displacement [m]
$t_o = \underline{0.05227}$	$x_o = \underline{0.0500}$
$t_f = \underline{0.08638}$	$x_f = \underline{0.1000}$

$$\langle v \rangle_2 = \frac{(0.100 \text{ m}) - (0.05 \text{ m})}{(0.08638 \text{ sec}) - (0.05227 \text{ sec})} = 1.46584 \frac{\text{m}}{\text{s}}$$

$$\boxed{\langle v \rangle_2 = 1.46584 \frac{\text{m}}{\text{s}}}$$

The average of the two sequential times is:

$$\langle t \rangle_2 = \frac{(0.08638 \text{ sec}) + (0.05227 \text{ sec})}{2} = 0.069325 \text{ sec}$$

$$\boxed{\langle t \rangle_2 = 0.069325 \text{ sec}}$$

For the third and fourth data points (to find the third average velocity):

Time [sec]	X-displacement [m]
$t_o = \underline{0.08638}$	$x_o = \underline{0.1000}$
$t_f = \underline{0.1149}$	$x_f = \underline{0.1500}$

$$\langle v \rangle_3 = \frac{(0.1500 \text{ m}) - (0.100 \text{ m})}{(0.1149 \text{ sec}) - (0.08638 \text{ sec})} = 1.75316 \frac{\text{m}}{\text{s}}$$

$$\boxed{\langle v \rangle_3 = 1.75316 \frac{\text{m}}{\text{s}}}$$

The average of the two sequential times is:

$$\langle t \rangle_3 = \frac{(0.1149 \text{ sec}) + (0.08638 \text{ sec})}{2} = 0.10064 \text{ sec}$$

$$\boxed{\langle t \rangle_3 = 0.10064 \text{ sec}}$$

For the fourth and fifth data points (to find the fourth average velocity):

Time [sec]	X-displacement [m]
$t_o = 0.1149$	$x_o = 0.1500$
$t_f = 0.1394$	$x_f = 0.2000$

$$\langle v \rangle_4 = \frac{(0.2000 \text{ m}) - (0.1500 \text{ m})}{(0.1394 \text{ sec}) - (0.1149 \text{ sec})} = 2.04082 \frac{\text{m}}{\text{s}}$$

$$\boxed{\langle v \rangle_4 = 2.04082 \frac{\text{m}}{\text{s}}}$$

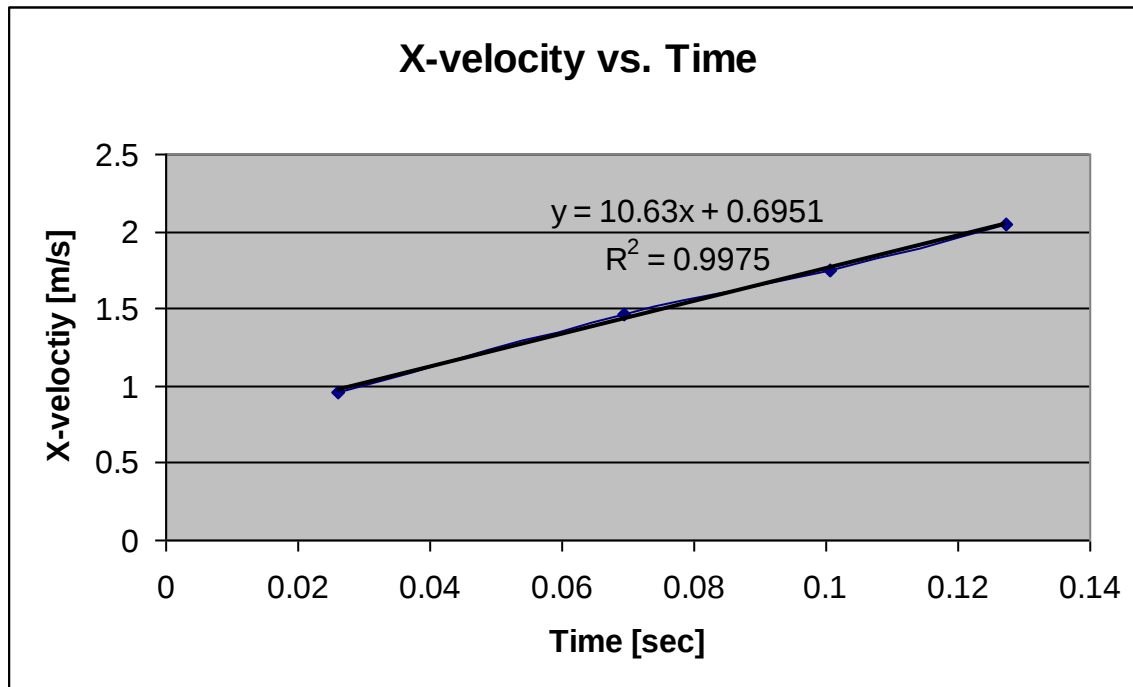
The average of the two sequential times is:

$$\langle t \rangle_4 = \frac{(0.1394 \text{ sec}) + (0.1149 \text{ sec})}{2} = 0.12715 \text{ sec}$$

$$\boxed{\langle t \rangle_4 = 0.12715 \text{ sec}}$$

If we consolidate the above four calculations into a single table, we get the following:

Time [sec]	X-velocity [m/s]
0.026135	0.95657
0.069325	1.46584
0.100640	1.75316
0.127150	2.04082



** Note that this is the same calculations performed by the Lab Pro software. **

** Also note that if we followed the same process yet again, we could find the average accelerations (and if we wanted to be super fancy, compare that to “g”). ☺ **

** Finally, note the values of the regression for each of the above graphs; by “calculating” the average velocities, there is a degradation of the “reliability” of the data. Hence, to get a more accurate value for the initial velocity (as we did for lab 5), it would be more appropriate to use the first graph of x-displacement versus time, rather than x-velocity versus time. **

If we compare the best fit line from the x-displacement versus time graph to the equation for kinematic motion:

$$x = \frac{1}{2} a_x t^2 + v_{o,x} t + x_o \quad \underline{\quad \quad \quad} \quad x = \left(5.3529 \frac{m}{s^2} \right) t^2 + \left(0.6909 \frac{m}{s} \right) t + (-0.0002 m)$$

a_x [m/s ²]	$v_{o,x}$ [m/s]	x_o [m]
<u>10.7058</u>	<u>0.6909</u>	<u>-0.0002</u>

So, it is obvious that for this problem, the x-direction is pointing toward the center of mass of the earth in this coordinate frame. (Typically, we’ve been calling this value “y”;

however, the axis naming is arbitrary – as long as we are consistent throughout the problem. [Technically, we could have called this the “nadir-axis”.])

5. $F = 3.5x$ is a linear relationship. What data would you plot along the y-axis, the x-axis?

Since the given equation is a linear relationship, it must be of the form:

$$\begin{aligned} y &= (m)x + b \\ F &= (-k)x + 0 \end{aligned}$$

- Along the y-axis, I would plot the measured force.
- I would label this axis: “Force [N]” or “Force [kg-m/s²]”.
- Along the x-axis, I would plot the measured displacement.
- I would label this axis: “Displacement [cm]” or “X [cm]”.

What is the slope?

The slope of the graph for $F = -3.5x$ would be 3.5. This represents a constant of how much the Force will change when we change the displacement.

What should be the value of the y-intercept?

If we examine the given equation and the equation for a linear relationship, we can easily determine the value of the y-intercept.

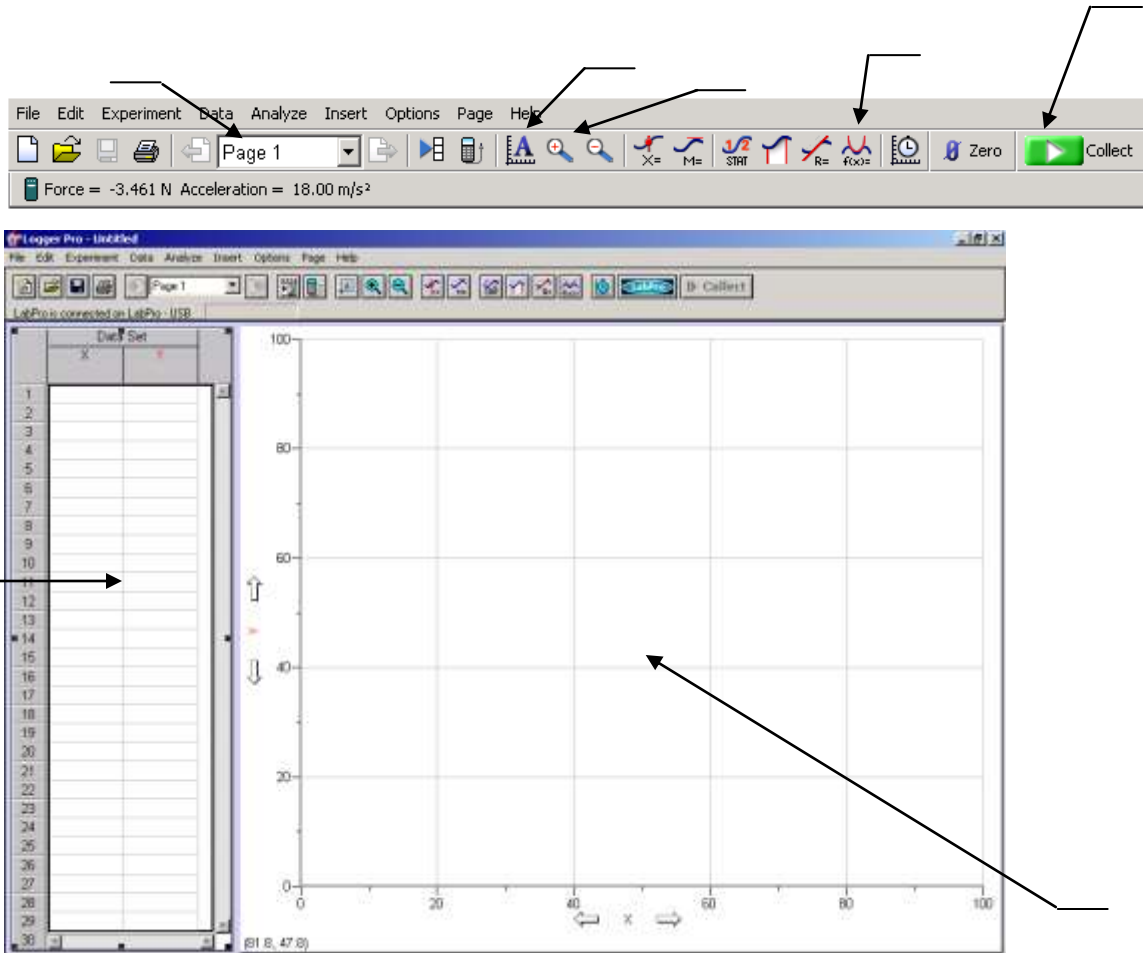
$$\begin{aligned} y &= (m)x + b \\ F &= (3.5)x + 0 \end{aligned}$$

The y-intercept is the value b in the equation for a linear relationship. This is the value of y , when $x = 0$. Thus for our given equation, by inspection, the y-intercept is 0.

By calculation:

$$\begin{aligned} F &= 3.5x \\ F &= 3.5(0) = 0 \end{aligned}$$

6. Match the descriptions with the correct buttons.



- A. Navigate through the program's different pages.
- B. Start the collection of data.
- C. Zoom in
- D. Tables where data is stored in spreadsheet format.
- E. Autoscale a graph.
- F. The place where the data is displayed graphically.
- G. Curve fitting

