

Gauss's Law

Name:

PES 2160 Prelab Questions

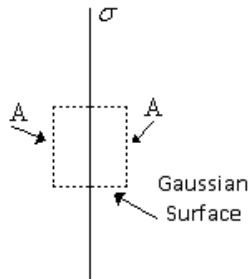
Lab Station:

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1. In Prelab questions 2 and 3, the electric fields predicted by Gauss' Law for the infinite planar and infinite cylindrical charge symmetries will be given to you. Using these results, you will recast the electric fields and charge densities as potentials. Explain why this recasting is necessary.

As we saw with the experiment where we plotted out the Electric Field using the carbon paper; it is very easy to measure the Electric Potential – but difficult to directly measure the Electric Field. We in turn used the plot of the Electric Potential lines to effectively draw in the Electric Field. Similarly, if we can recast everything in terms of the Electric Potential, it will make the experiment easier to perform and compare to a theoretical value.

2. Consider an infinite plane with a uniform surface charge density of σ .

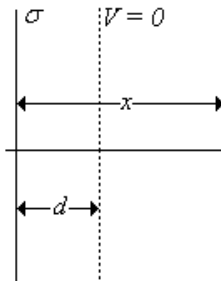


From Gauss' Law, the electric field of an infinite plane is:

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

The uniformly charged infinite plane has equipotential surfaces that are parallel to it.

Since we can pick anywhere we want to be zero potential, we will call the equipotential at $x = d$ the $V = 0$ equipotential, as shown in the figure ~~at right~~ below:



- a.) Using the relationship between potential and electric field:

$$\Delta V = - \int_d^x \vec{E} \cdot d\vec{s}$$

find the potential difference between the points $x = d$ and $x = x$.

First, let's plug in the electric field we found above and the infinitesimal displacement (pointing in the same direction as the electric field) [IMPORTANT: Note that if we went with the bounds of $x \rightarrow d$ for the integral, instead of $d \rightarrow x$ (as we have) ... then

the displacement vector wouldn't have the same sign and would be pointing in the opposite direction as the electric field! This would result in a -1 for the dot product; but still gives us the same answer. 😊:

$$\Delta V = - \int_d^x \frac{\sigma}{2\epsilon_0} dn (\hat{n} \bullet \hat{n}) = - \frac{\sigma}{2\epsilon_0} \int_d^x dn (1) = - \frac{\sigma}{2\epsilon_0} \int_d^x dn$$

Next, the trick is to use the negative sign to “flip the ratio”:

$$\Delta V = - \frac{\sigma}{2\epsilon_0} (n|_d^x = - \frac{\sigma}{2\epsilon_0} (x - d) = \frac{\sigma}{2\epsilon_0} (d - x)$$

This means we can determine the potential at any arbitrary distance x from the surface of the charged plane.

$$V(x) = - \frac{\sigma}{2\epsilon_0} (x - d)$$

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b.) The potential you found in part (a) has the form:

$$V = mx + b$$

where m and b depend on σ . Use the fact that at $x = 0$, $V = V_0$ to find m and b in terms of V_0 instead of σ . In this lab, you will compare the theoretical values of m and b to the experimentally measured values of m and b .

It's easy to see that the form mentioned here and the form we found do match, where:

$$V = mx + b = \left(- \frac{\sigma}{2\epsilon_0} \right) x + \left(\frac{\sigma d}{2\epsilon_0} \right)$$

$$m = - \frac{\sigma}{2\epsilon_0} \quad \text{and} \quad b = \frac{\sigma d}{2\epsilon_0}$$

If we apply the boundary condition at $x = 0$:

$$V(0) = \frac{\sigma}{2\epsilon_0} (d - 0) = \frac{\sigma d}{2\epsilon_0} = V_0$$

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Now, solve for $\frac{\sigma}{2\epsilon_0}$ and plug this back into our generic $V(x)$ equation:

$$\frac{\sigma}{2\epsilon_0} = \frac{V_0}{d}$$

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$$V(x) = \left(-\frac{V_0}{d}\right)x + (V_0)$$

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This means, now:

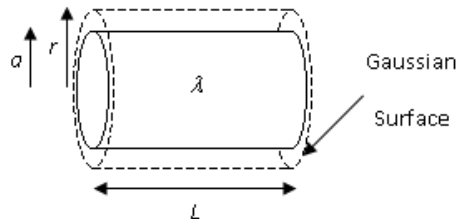
$$m = -\frac{V_0}{d} \text{ and } b = V_0$$

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We have now successfully recast the electric field of an infinite charged plane into potential (something we can measure and compare in the lab).

3. Consider an infinite cylinder of radius a with a uniform charge per length L of λ .

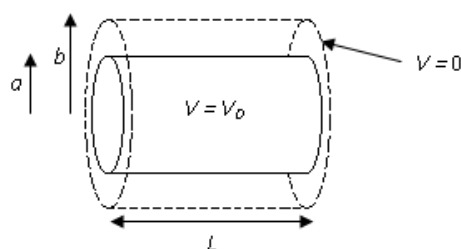
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From Gauss's Law, the electric field of an infinite cylinder at a radial distance r from its center is:

$$\vec{E} = \frac{2k\lambda}{r} \hat{r}$$

The uniformly charged infinite cylinder has equipotential surfaces that are also cylinders and concentric with the charged cylinder. We will call the equipotential at $r = b$ the $V(b) = 0$ and at $r = a$ the $V(a) = V_o$:



a.) Using the relationship between potential and electric field:

$$\Delta V = - \int_R^b \vec{E} \cdot d\vec{s}$$

find the potential difference between the points $r = b$ and $r = R$.

First, let's plug in the electric field we found above and the infinitesimal displacement (pointing in the same direction as the electric field) **IMPORTANT: Note that if we went with the bounds of $b \rightarrow R$ for the integral, instead of $R \rightarrow b$ (as we have) ... then the displacement vector wouldn't have the same sign and would be pointing in the opposite direction as the electric field! This would result in a -1 for the dot product; but still gives us the same answer. ☺**:

$$\Delta V = - \int_R^b \frac{2k\lambda}{r} dr (\hat{r} \cdot \hat{r}) = -2k\lambda \int_R^b \frac{1}{r} dr (1) = -2k\lambda \int_R^b \frac{1}{r} dr$$

Next, the trick is to use the negative sign to "flip the log":

$$\Delta V = -2k\lambda (\ln(r)) \Big|_R^b = -2k\lambda (\ln(b) - \ln(R)) = 2k\lambda (\ln(R) - \ln(b)) = 2k\lambda \ln\left(\frac{R}{b}\right)$$

This means we can determine the potential at any arbitrary distance R from the surface of the charged rod.

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$$V(R) = -\int_R^b \frac{2k\lambda}{r} dr = 2k\lambda \ln\left(\frac{R}{b}\right)$$

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b.) The potential you found in part a has the form:

$$V = A \ln(Br)$$

where A depends on λ and B is a geometric parameter. Use the fact that at $r = a$, $V(a) = V_o$ to find A and B in terms of V_o , a and b instead of λ . In this lab, you will compare the theoretical values of A and B to the experimentally measure values of A and B .

It's easy to see that the form mentioned here and the form we found do match, where:

$$V = A \ln(Br) = 2k\lambda \ln\left(\frac{R}{b}\right)$$

$$A = 2k\lambda \text{ and } B = \frac{1}{b}$$

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If we apply the boundary condition at $R = a$:

$$V(a) = 2k\lambda \ln\left(\frac{a}{b}\right) = V_o$$

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Now, solve for $2k\lambda$ and plug this back into our generic $V(R)$ equation:

$$2k\lambda = \frac{V_o}{\ln\left(\frac{a}{b}\right)}$$

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$$V(R) = A \ln(Br) = \left(\frac{V_o}{\ln\left(\frac{a}{b}\right)} \right) \ln\left(\frac{R}{b}\right)$$

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This means, now:

$$A = \frac{V_o}{\ln\left(\frac{a}{b}\right)} \quad \text{and} \quad B = \frac{1}{b}$$

We have now successfully recast the electric field of an infinite charged cylinder into potential (something we can measure and compare in the lab).

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