Experiment #5 Capacitors Pre-lab Questions

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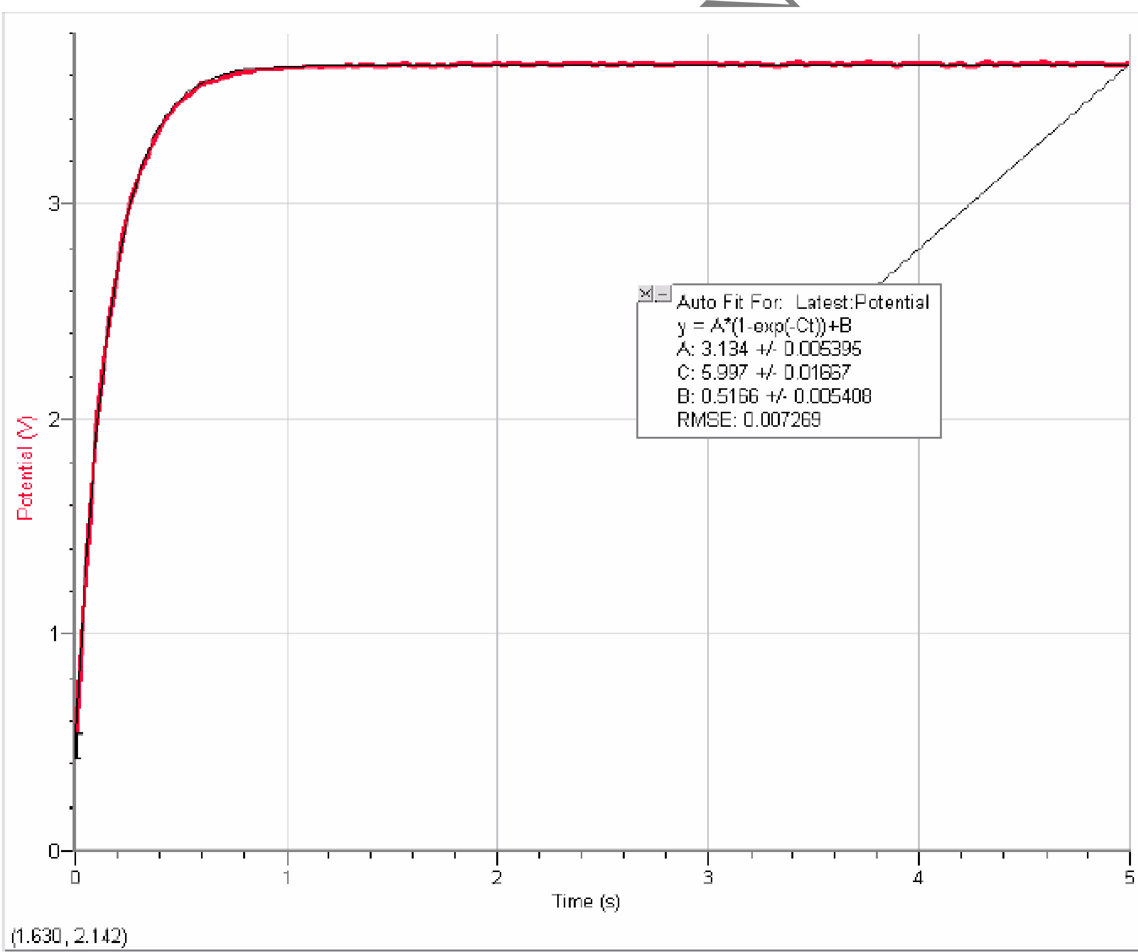
- **Examine the following graph of an RC circuit using Logger Pro:**
  - **Determine τ**
  - **Calculate the capacitance if the resistor has a value of 400kΩ.**



Notice that the above graph is an exponential. Using the Logger Pro software, the following best-fit line was found.

$$y = 3.134\ V\left(1 - e^{(-(5.997\ Hz)t)}\right) + 0.5166\ V$$

The root mean square error for the fit was 0.007269, which means that the regression of the fit is 0.992731. This means that the fit is nearly a perfect exponential saturation curve.

For an RC circuit, the potential as a function of time across a capacitor being charged is given as:

$$V(t) = V_f\left(1 - e^{\left(-\frac{t}{\tau}\right)}\right) + V_o = V_f\left(1 - e^{\left(-\frac{t}{RC}\right)}\right) + V_o$$

In the equation above, the value τ (tao) [don't confuse this with volume or torque ☺] is a value called the "time constant". This is the duration of time which it takes for the capacitor to reach 63% of its saturation value. If we compare the best-fit line with the equation for the potential drop across a charging capacitor, we can see the direct similarities.

$$y = 3.134\ V\left(1 - e^{(-(5.997\ Hz)t)}\right) + 0.5166\ V$$

$$V(t) = V_f\left(1 - e^{\left(-\frac{t}{\tau}\right)}\right) + V_o$$

It is obvious that the number in the exponential is:

$$-\frac{t}{\tau} = -(5.997\ Hz)t$$

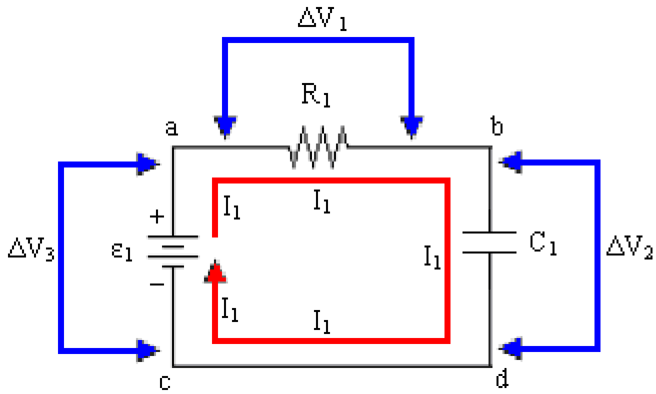
Solving this for tao, we see (the units of Hertz are 1/sec):

$$\boxed{\tau = \frac{1}{5.997\ Hz} = 0.16675\ \text{sec}}$$

Thus:

V_f	τ	V_o
3.134 [V]	0.16675 [sec]	0.5166 [V]

The value of the "time constant" is interesting, because it is directly related to the value of the capacitor being charged and the resistance in the RC circuit.



$$\tau = RC$$

So, if the circuit above had a resistor of resistance 400 kΩ, we could easily use the value we found from the best-fit line for the time constant and calculate the capacitance of the capacitor.

$$\tau = 0.16675 \text{ sec} = RC$$

$$(400\ k\Omega)C = 0.16675 \text{ sec}$$

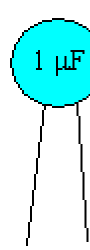
$$C = \frac{0.16675 \text{ sec}}{400 \times 10^3\ \Omega} = 4.16875 \times 10^{-7}\ \frac{\text{sec}}{\Omega}$$

$$\boxed{C = 4.16875 \times 10^{-7}\ F}$$

** Note that the units of capacitance are [sec/Ω] which is called a "Farad" [F]. **

** Aside **

We could easily customize the time constant for an RC circuit by picking "good" values for the resistor and the capacitor in the circuit. For example, say that we were not using a computer (that could accurately measure on the order of partial seconds), but rather a stop watch for this experiment. We would prefer the time constant to be on the order of seconds, rather than milliseconds. Assume we have a 1 microfarad capacitor. This would likely look like a blue circle with 1 μF written on it.



$$\tau = 1.0 \text{ sec} = RC$$

$$R(1\ \mu F) = 1.0 \text{ sec}$$

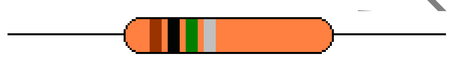
$$R = \frac{1.0 \text{ sec}}{1.0 \times 10^{-6}\ \frac{\text{sec}}{\Omega}} = 1.0 \times 10^{6}\ \Omega$$

$$R = 1.0 \times 10^{6}\ \Omega = 1\ M\Omega$$

$$10 \text{ x } 10^{5} \pm 10\%\ \Omega$$

$$(1)\ (0) \text{ x } 10^{(5)} \pm (10\%)\ \Omega$$

Brown – Black – Green – Silver



So, if we built the RC circuit above with the 1 MΩ resistor and the 1 μF capacitor, we could effectively get a time constant on the order of 1 second, which is easily measurable with a stopwatch.

** End Aside **