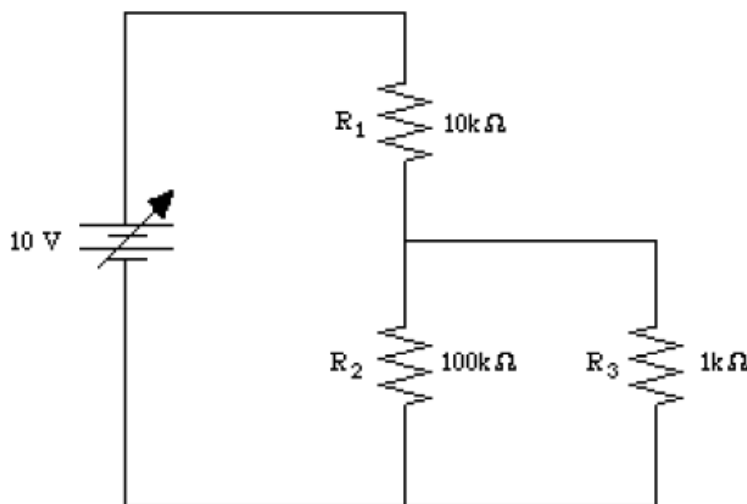


Experiment #8 Kirchhoff's Laws Pre-lab Questions

**** Disclaimer: This pre-lab is *not* to be copied, in whole or in part, unless a proper reference is made as to the source. (It is strongly recommended that you use this document *only* to generate ideas, or as a reference to explain complex physics necessary for completion of your work.) Copying of the contents of this web site and turning in the material as “original material” is *plagiarism* and will result in serious consequences as determined by your instructor. These consequences may include a failing grade for the particular pre-lab or a failing grade for the entire semester, at the discretion of your instructor. ****

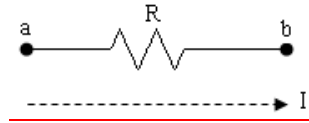
- Calculate all the voltages and currents for all the components. You will need to use the loop and junction rules.



This lab and pre-lab will make extensive use of Ohm's Law, so it is of benefit to take a moment to review a little about how Ohm's Law works and set-up an attack plan for solving problems using Kirchhoff's Rules. [Note: The correct spelling of this dead physicist's name is Gustav Robert Kirchhoff, and his last name is spelt with two h's and two f's. (If you're interested, it is pronounced: "KIRK-cough".) ** I will admit that sometimes in English textbooks it is spelled "Kirchoff", but that is not the correct German spelling of the name. It's the same as my ancestors, whose name was "Behr". When they came to America (a long time ago) and settled in Amish Pennsylvania (It is interesting to note that these people are commonly called "Pennsylvania Dutch", even though they are not Dutch, but German. The reason is that in German, the word for German is "deutsch", which sounds like "dutch".), it was changed to "Bair", "Baer", "Bayer", "Blair", and "Bear", because some immigration officer didn't know his

German. Could you imagine me winning the Nobel Prize and it having “William L. Bayer, Jr.” written on it? I’d be pissed. ☺ ** Anyway back to Ohm’s Law.]

Assume we have the following configuration – we have a piece of wire with a resistor (R) and a current moving through the wire (I):



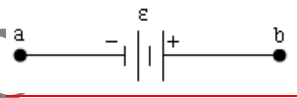
Case 1: If we traverse this path from a to b (in the same direction of the current) while going around a loop, then the potential drop is given by Ohm’s Law as:

$$\Delta V = -IR$$

Case 2: If we traverse this path from b to a (in the opposite direction of the current) while going around a loop, then the potential rise is given by Ohm’s Law as:

$$\Delta V = +IR$$

Next, assume we have the following configuration – we have a piece of wire with a electromagnetic force (EMF, $\mathcal{E}$) [this is also sometime referred to as a “battery” or a “power supply”]:



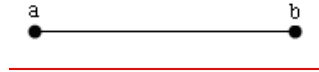
Case 1: If we traverse this path from a to b (from the negative side of the EMF to the positive side of the EMF) while going around a loop, then the potential rise is given by Ohm’s Law as:

$$\Delta V = +\mathcal{E}$$

Case 2: If we traverse this path from b to a (from the positive side of the EMF to the negative side of the EMF) while going around a loop, then the potential decrease is given by Ohm’s Law as:

$$\Delta V = -\mathcal{E}$$

Finally, assume that we have the following configuration – we have a piece of wire with nothing on it (just a plain piece of wire):



Case 1: If we traverse this path from a to b while going around a loop, then the potential rise is given by Ohm's Law as:

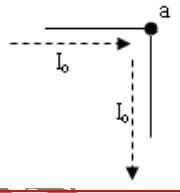
$$\underline{\Delta V = 0}$$

Case 2: If we traverse this path from b to a while going around a loop, then the potential decrease is given by Ohm's Law as:

$$\underline{\Delta V = 0}$$

Kirchhoff's Zeroeth Rule (Corner Rule):

The current does not change around a corner in a circuit diagram.



At corner a, the current does not change magnitude.

Kirchhoff's First Rule (Loop Rule):

The sum of the potential rises and decreases around a closed loop must be zero.

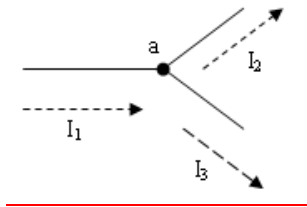
$$\underline{\sum_{i=1}^n \Delta V_i = 0}$$

Kirchhoff's Second Rule (Junction Rule):

The sum of the currents into a junction point must equal the sum of the current out of a junction point.

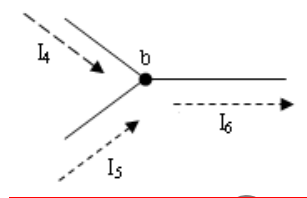
$$\underline{\sum_{j=1}^n I_{In,j} = \sum_{k=1}^m I_{Out,k}}$$

Examples:



At the junction point a, there is a single current going into the junction (I_1), and there are two currents leaving the junction (I_2 and I_3). Thus, according to Kirchhoff's Second Rule:

$$I_1 = I_2 + I_3$$



At the junction point b, there are two currents going into the junction (I_4 and I_5), and there is a single current leaving the junction (I_6). Thus, according to Kirchhoff's Second Rule:

$$I_4 + I_5 = I_6$$

***** THAT'S IT! *****

Problem Solving Techniques (PST) for Kirchhoff's Rules Problems:

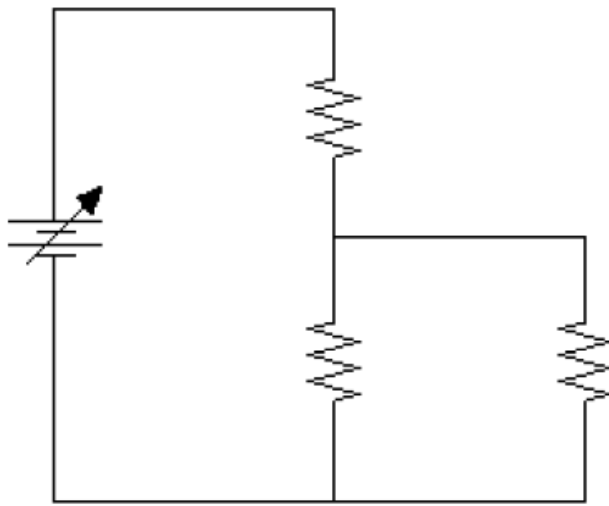
1. Draw the circuit diagram with all of its components – including resistors, electromagnetic forces, capacitors, inductors, transistors, etc...
2. Label all of the components clearly with standard notation – label resistors as R_1 , R_2 , etc...; label EMF's as $\mathcal{E}_1$, $\mathcal{E}_2$, etc...; label capacitors as C_1 , C_2 , etc ...; and so forth...
3. Mark the positive side of each of the EMF's with a "+", as well as the negative side of each of the EMF's with a "-".
4. Label each corner and junction of the circuit diagram with a lower case letter of the alphabet (probably a good idea to start with "a" and go alphabetically).
5. Draw in currents. The direction can be arbitrary, so just pick a direction *and stick with it till the end of the problem!* Make sure that at junctions you have the correct number of currents though each piece of wire.
6. Label the current clearly with the values I_1 , I_2 , I_3 , etc...
7. Draw in the potential drops and rises across your components.
8. Label the potential drops and rises with ΔV_1 , ΔV_2 , ΔV_3 , etc ...

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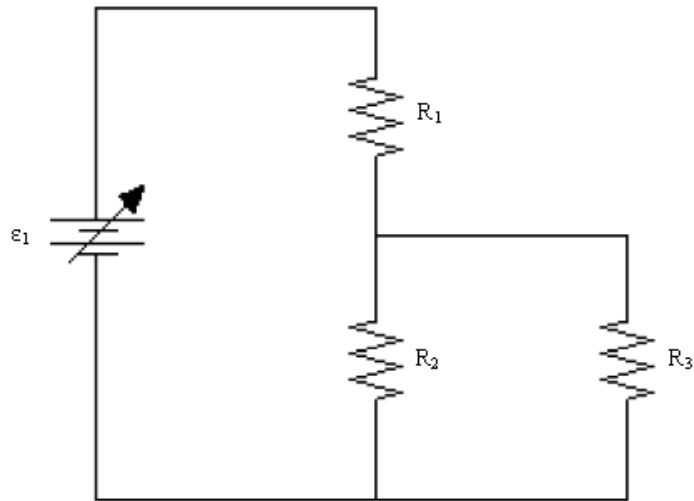
9. Apply the Loop Rule for each closed loop in the circuit diagram (use Ohm's Law as described at the beginning of this pre-lab).
10. Apply the Junction Rule for each junction in the circuit diagram.
11. Solve the system of equations for the knowns and unknowns.

Let's apply the PST's and all of Kirchhoff's Rules and Ohm's Law to the circuit for this pre-lab.

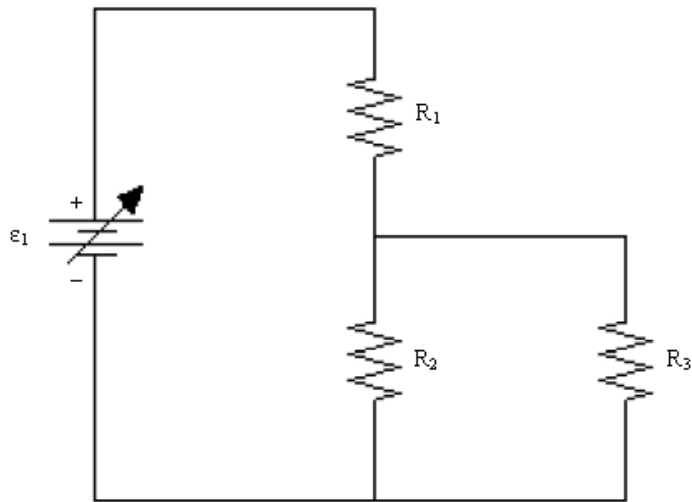
PST 1- Draw the circuit diagram with all of its components – including resistors, electromagnetic forces, capacitors, inductors, transistors, etc...:



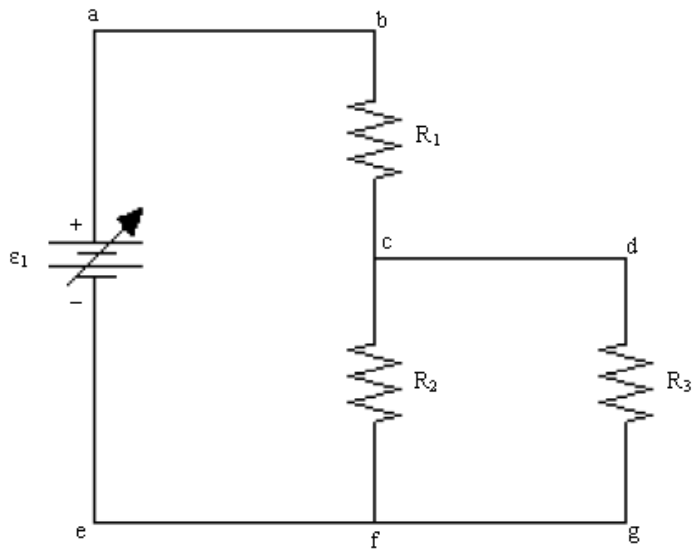
PST 2 - Label all of the components clearly with standard notation – label resistors as R_1 , R_2 , etc...; label EMF's as $\mathcal{E}_1$, $\mathcal{E}_2$, etc...; label capacitors as C_1 , C_2 , etc ...; and so forth ...:



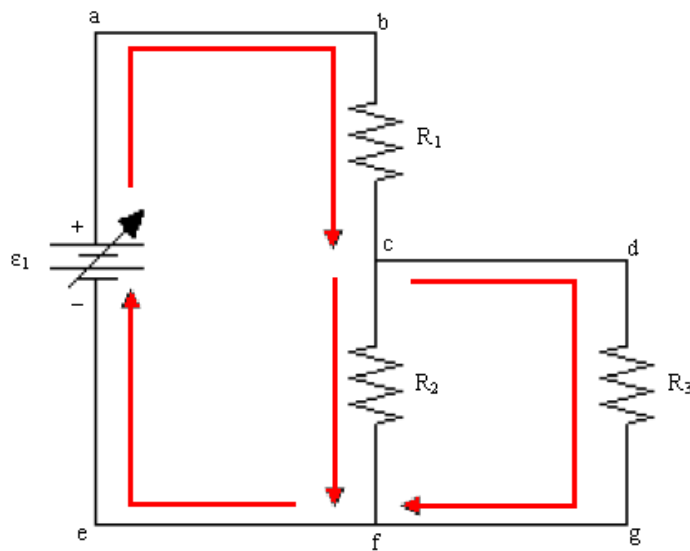
PST 3 - Mark the positive side of each of the EMF's with a "+", as well as the negative side of each of the EMF's with a "-".



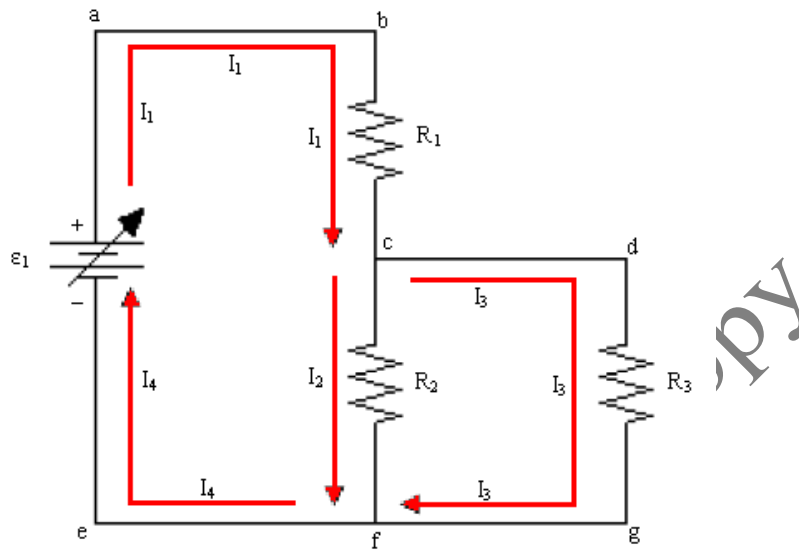
PST 4 - Label each corner and junction of the circuit diagram with a lower case letter of the alphabet (probably a good idea to start with "a" and go alphabetically):



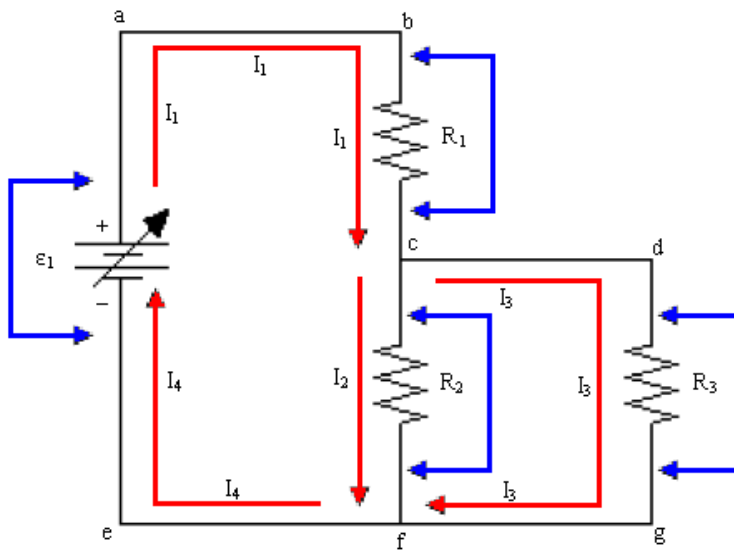
PST 5 - Draw in currents. The direction can be arbitrary, so just pick a direction *and stick with it till the end of the problem!* Make sure that at junctions you have the correct number of currents through each piece of wire.



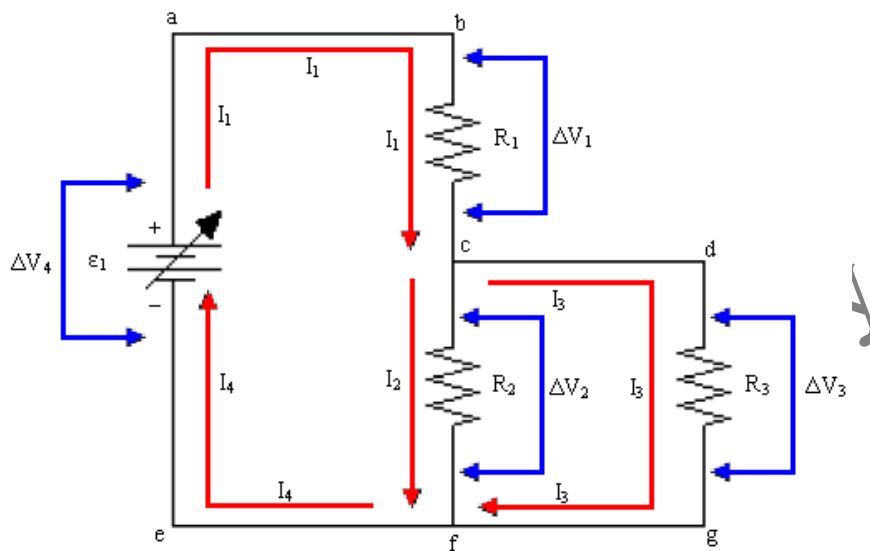
PST 6 - Label the current clearly with the values I_1 , I_2 , I_3 , etc... (Notice we already applied Kirchhoff's Zeroeth Rule):



PST 7 - Draw in the potential drops and rises across your components:



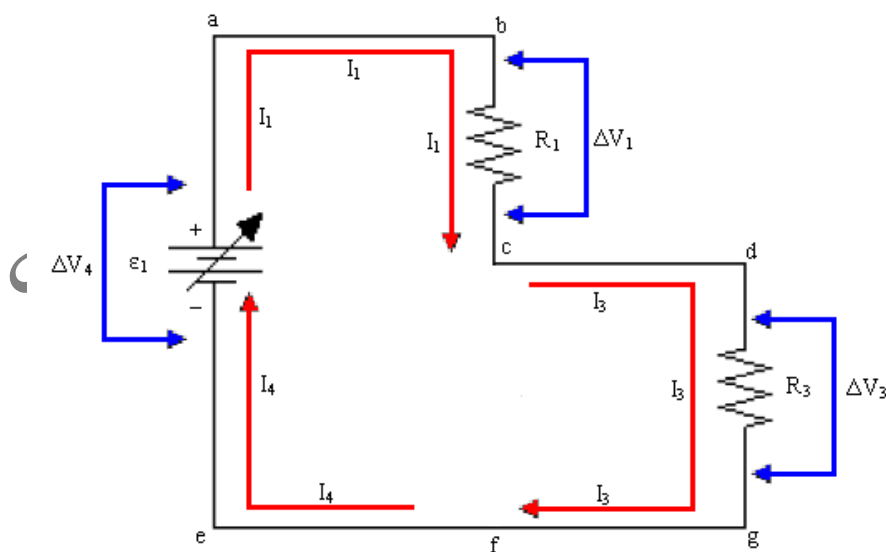
PST 8 - Label the potential drops and rises with ΔV_1 , ΔV_2 , ΔV_3 , etc:



PST 9 - Apply the Loop Rule for each closed loop in the circuit diagram (use Ohm's Law as described at the beginning of this pre-lab):

We have 3 loops that we need to consider each individually. These three loops are (using the letters to describe the loops):

Loop 1 = a-b-c-d-g-f-e-a:



** NOTE: we did not “get rid” of the wire connecting c to f, we just are “selectively ignoring” it at the moment since it plays no part in the loop we are concerned with. **

Applying Ohm’s Law using the convention set up at the beginning of the pre-lab:

From a to b:

$$\underline{\Delta V = 0}$$

From b to c:

$$\underline{\Delta V_1 = -I_1 R_1}$$

(** NOTICE THAT IT IS NEGATIVE SINCE WE ARE TRAVERSING IN THE SAME DIRECTION AS THE CURRENT! **)

From c to d:

$$\underline{\Delta V = 0}$$

From d to g:

$$\underline{\Delta V_3 = -I_3 R_3}$$

From g to f:

$$\underline{\Delta V = 0}$$

From f to e:

$$\underline{\Delta V = 0}$$

From e to a:

$$\underline{\Delta V_4 = +\mathcal{E}}$$

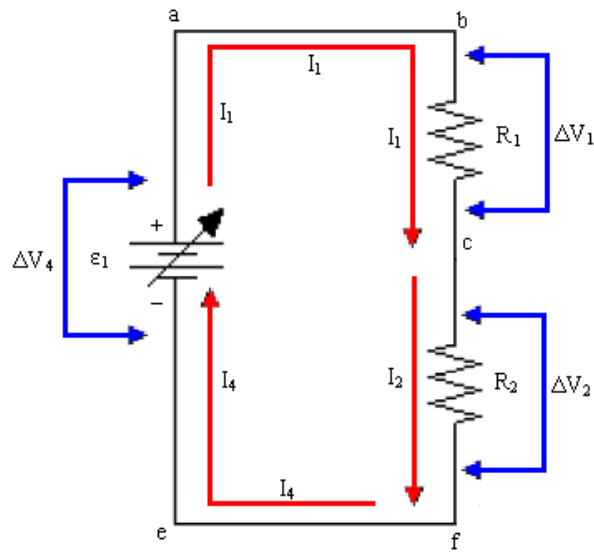
According to the loop rule, the sum of the potential rises and decreases around a closed loop must be zero.

$$\underline{\sum_{i=1}^n \Delta V_i = 0}$$

$$\underline{\sum_{i=1}^7 \Delta V_i = 0 - I_1 R_1 + 0 - I_3 R_3 + 0 + 0 + \mathcal{E} = 0}$$

$$\underline{-I_1 R_1 - I_3 R_3 + \mathcal{E} = 0 \text{ (eq. 1)}}$$

Loop 2 = a-b-c-f-e-a:



Applying Ohm's Law using the convention set up at the beginning of the pre-lab:

From a to b:

$$\underline{\Delta V = 0}$$

From b to c:

$$\underline{\Delta V_1 = -I_1 R_1}$$

From c to f:

$$\underline{\Delta V_2 = -I_2 R_2}$$

From f to e:

$$\underline{\Delta V = 0}$$

From e to a:

$$\underline{\Delta V_4 = +\varepsilon}$$

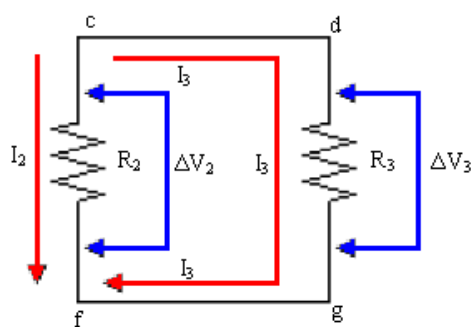
According to the loop rule, the sum of the potential rises and decreases around a closed loop must be zero.

$$\underline{\sum_{i=1}^n \Delta V_i = 0}$$

$$\underline{\sum_{i=1}^5 \Delta V_i = 0 - I_1 R_1 - I_2 R_2 + 0 + \varepsilon = 0}$$

$$\underline{-I_1 R_1 - I_2 R_2 + \varepsilon = 0 \text{ (eq. 2)}}$$

Loop 3 = c-d-g-f-c:



Applying Ohm's Law using the convention set up at the beginning of the pre-lab:

From c to d:

$$\underline{\Delta V = 0}$$

From d to g:

$$\underline{\Delta V_3 = -I_3 R_3}$$

From g to f:

$$\underline{\Delta V = 0}$$

From f to c:

$$\underline{\Delta V_2 = +I_2 R_2}$$

(** NOTICE THAT IT IS POSTIVE SINCE WE ARE TRAVERSING IN THE OPPOSITE DIRECTION THAN THE CURRENT! **)

According to the loop rule, the sum of the potential rises and decreases around a closed loop must be zero.

$$\underline{\sum_{i=1}^n \Delta V_i = 0}$$

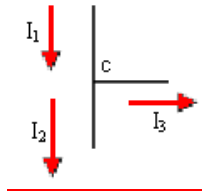
$$\underline{\sum_{i=1}^4 \Delta V_i = 0 - I_3 R_3 + 0 + I_2 R_2 = 0}$$

$$\underline{-I_3 R_3 + I_2 R_2 = 0 \text{ (eq. 3)}}$$

PST 10 - Apply the Junction Rule for each junction in the circuit diagram:

We have 2 junctions that we need to consider each individually. These two junctions are (using the letters to describe the junctions):

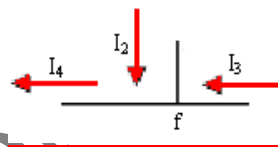
Junction 1 @ c:



According to the junction rule, the sum of the currents into a junction point must equal the sum of the current out of a junction point.

$$\underline{I_1 = I_2 + I_3 \text{ (eq. 4)}}$$

Junction 1 @ f:



According to the junction rule, the sum of the currents into a junction point must equal the sum of the current out of a junction point.

$$\underline{I_2 + I_3 = I_4 \text{ (eq.5)}}$$

PST 11 - Solve the system of equations for the knowns and unknowns:

We are given the following information from the beginning of the problem:

$\underline{\varepsilon}$	$\underline{R_1}$	$\underline{R_2}$	$\underline{R_3}$
$\underline{10 \text{ V}}$	$\underline{10 \text{ k}\Omega}$	$\underline{100 \text{ k}\Omega}$	$\underline{1 \text{ k}\Omega}$

That means that the unknowns are:

$$\underline{I_1, I_2, I_3, \text{ and } I_4}$$

Here is the list of consolidated equations we found using the loop rule and junction rule on the circuit:

$$\underline{-I_1 R_1 - I_3 R_3 + \varepsilon = 0 \text{ (eq. 1)}}$$

$$\underline{-I_1 R_1 - I_2 R_2 + \varepsilon = 0 \text{ (eq. 2)}}$$

$$\underline{-I_3 R_3 + I_2 R_2 = 0 \text{ (eq. 3)}}$$

$$\underline{I_1 = I_2 + I_3 \text{ (eq. 4)}}$$

$$\underline{I_2 + I_3 = I_4 \text{ (eq. 5)}}$$

First, start by solving equation 3 for I_3 :

$$\underline{I_3 = \frac{I_2 R_2}{R_3} \text{ (eq. 6)}}$$

Plug equation 6 into equation 1:

$$\begin{aligned} & \underline{-I_1 R_1 - \left(\frac{I_2 R_2}{R_3} \right) R_3 + \varepsilon = 0} \\ & \underline{-I_1 R_1 - I_2 R_2 + \varepsilon = 0} \end{aligned}$$

Notice that this is exactly the same as equation 2! So at least we did the loop rules right!
☺

Look at equation 4 and equation 5. Notice that the sums are the same for both of those equations. This must mean that:

$$\underline{I_1 = I_4 \text{ (eq. 7)}}$$

Use equation 4 and plug the value for I_1 into equation 2:

$$\begin{aligned} & \underline{-(I_2 + I_3)R_1 - I_2 R_2 + \varepsilon = 0} \\ & \underline{-I_2 R_1 - I_3 R_1 - I_2 R_2 + \varepsilon = 0 \text{ (eq. 8)}}$$

Use equation 6 and plug in the value of I_3 into equation 8, then solve for I_2 :

$$-I_2 R_1 - \left(\frac{I_2 R_2}{R_3} \right) R_1 - I_2 R_2 + \varepsilon = 0$$

$$-I_2 \left(R_1 + \frac{R_1 R_2}{R_3} + R_2 \right) = -\varepsilon$$

$$I_2 = \frac{\varepsilon}{\left(R_1 + \frac{R_1 R_2}{R_3} + R_2 \right)}$$

$$I_2 = \frac{\varepsilon(R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)}$$

Use the value we just found for I_2 and plug it into equation 3, then solve for I_3 :

$$-I_3 R_3 + \left(\frac{\varepsilon(R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)} \right) R_2 = 0$$

$$-I_3 R_3 = - \frac{\varepsilon(R_2 R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)}$$

$$I_3 = \frac{\varepsilon(R_2)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)}$$

Use the values we have found for I_2 and for I_3 and plug them into equation 4, then solve for I_1 :

$$I_1 = I_2 + I_3 = \frac{\varepsilon(R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)} + \frac{\varepsilon(R_2)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)}$$

$$I_1 = \frac{\varepsilon(R_3) + \varepsilon(R_2)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)}$$

$$I_1 = \frac{\varepsilon(R_2 + R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)}$$

Use the value we have found for I_1 and plug it into equation 7, then solve for I_4 :

$$\underline{I_1 = I_4}$$

$$I_4 = \frac{\varepsilon(R_2 + R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)}$$

Now we have expressions for all four currents in the original circuit.

$$\begin{aligned} I_1 &= \frac{\varepsilon(R_2 + R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)} & I_2 &= \frac{\varepsilon(R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)} \\ I_3 &= \frac{\varepsilon(R_2)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)} & I_4 &= \frac{\varepsilon(R_2 + R_3)}{(R_1 R_3 + R_1 R_2 + R_2 R_3)} \end{aligned}$$

We can now plug in the value of the given information and solve for the numerical value of each of the currents.

$$I_1 = \frac{10 \text{ V}(100\text{k}\Omega + 1\text{k}\Omega)}{((10\text{k}\Omega)(1\text{k}\Omega) + (10\text{k}\Omega)(100\text{k}\Omega) + (100\text{k}\Omega)(1\text{k}\Omega))}$$

$$I_1 = \frac{10 \text{ V}(101000\Omega)}{((1 \times 10^7 \Omega^2) + (1 \times 10^9 \Omega^2) + (1 \times 10^8 \Omega^2))} = \frac{10 \text{ V}(101000\Omega)}{(1110000000\Omega^2)}$$

$$I_1 = 9.0991 \times 10^{-4} \text{ A} = 0.90991 \text{ mA}$$

$$I_2 = \frac{10 \text{ V}(1\text{k}\Omega)}{((10\text{k}\Omega)(1\text{k}\Omega) + (10\text{k}\Omega)(100\text{k}\Omega) + (100\text{k}\Omega)(1\text{k}\Omega))}$$

$$I_2 = \frac{10 \text{ V}(1000\Omega)}{((1 \times 10^7 \Omega^2) + (1 \times 10^9 \Omega^2) + (1 \times 10^8 \Omega^2))} = \frac{10 \text{ V}(1000\Omega)}{(1110000000\Omega^2)}$$

$$I_2 = 9.009 \times 10^{-6} \text{ A} = 9.009 \mu\text{A}$$

$$I_3 = \frac{10 \text{ V}(100\text{k}\Omega)}{((10\text{k}\Omega)(1\text{k}\Omega) + (10\text{k}\Omega)(100\text{k}\Omega) + (100\text{k}\Omega)(1\text{k}\Omega))}$$

$$I_1 = \frac{10 \text{ V}(100000\Omega)}{((1 \times 10^7 \Omega^2) + (1 \times 10^9 \Omega^2) + (1 \times 10^8 \Omega^2))} = \frac{10 \text{ V}(100000\Omega)}{(1110000000\Omega^2)}$$

$$\boxed{I_3 = 9.009 \times 10^{-4} \text{ A} = 0.9009 \text{ mA}}$$

$$I_4 = \frac{10 \text{ V}(100 \text{ k}\Omega + 1 \text{ k}\Omega)}{((10 \text{ k}\Omega)(1 \text{ k}\Omega) + (10 \text{ k}\Omega)(100 \text{ k}\Omega) + (100 \text{ k}\Omega)(1 \text{ k}\Omega))}$$

$$I_4 = \frac{10 \text{ V}(101000 \Omega)}{((1 \times 10^7 \Omega^2) + (1 \times 10^9 \Omega^2) + (1 \times 10^8 \Omega^2))} = \frac{10 \text{ V}(101000 \Omega)}{(1110000000 \Omega^2)}$$

$$\boxed{I_4 = 9.0991 \times 10^{-4} \text{ A} = 0.90991 \text{ mA}}$$

Finally, we can determine the potentials (voltages) across each of the components by Ohm's Law.

$$\Delta V_1 = -I_1 R_1$$

$$\Delta V_1 = -(0.90991 \text{ mA})(10 \text{ k}\Omega) = -(0.90991 \times 10^{-3} \text{ A})(1 \times 10^4 \Omega)$$

$$\boxed{\Delta V_1 = -9.0991 \text{ V}}$$

$$\Delta V_2 = -I_2 R_2$$

$$\Delta V_2 = -(9.009 \mu\text{A})(100 \text{ k}\Omega) = -(9.009 \times 10^{-6} \text{ A})(1 \times 10^5 \Omega)$$

$$\boxed{\Delta V_2 = -0.9009 \text{ V}}$$

$$\Delta V_3 = -I_3 R_3$$

$$\Delta V_3 = -(0.9009 \text{ mA})(1 \text{ k}\Omega) = -(0.9009 \times 10^{-3} \text{ A})(1 \times 10^3 \Omega)$$

$$\boxed{\Delta V_3 = -0.9009 \text{ V}}$$

$$\boxed{\Delta V_4 = \mathcal{E} = 10 \text{ V}}$$

<NOTE: If you are familiar with Linear Algebra and Matrix Notation, you can enter all four equations (eq. 1, eq. 2, eq. 3, and eq. 5) into an augmented matrix.>

$$\begin{bmatrix} -R_1 & 0 & -R_3 & 0 \\ -R_1 & -R_2 & 0 & 0 \\ 0 & R_2 & R_3 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -\varepsilon \\ -\varepsilon \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} -R_1 & 0 & -R_3 & 0 & -\varepsilon \\ -R_1 & -R_2 & 0 & 0 & -\varepsilon \\ 0 & R_2 & R_3 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 \end{array} \right]$$

Using Gauss-Jordan Elimination techniques to get the augmented matrix into reduced row echelon form, you get the following:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \varepsilon(R_2 + R_3)/(R_1R_3 + R_1R_2 + R_2R_3) \\ 0 & 1 & 0 & 0 & \varepsilon(R_2)/(R_1R_3 + R_1R_2 + R_2R_3) \\ 0 & 0 & 1 & 0 & \varepsilon(R_3)/(R_1R_3 + R_1R_2 + R_2R_3) \\ 0 & 0 & 0 & 1 & \varepsilon(R_2 + R_3)/(R_1R_3 + R_1R_2 + R_2R_3) \end{array} \right]$$

Which means (note that this is the same result we obtained from back-substitution and algebra above):

$$I_1 = \frac{\varepsilon(R_2 + R_3)}{(R_1R_3 + R_1R_2 + R_2R_3)} \quad I_2 = \frac{\varepsilon(R_3)}{(R_1R_3 + R_1R_2 + R_2R_3)}$$

$$I_3 = \frac{\varepsilon(R_2)}{(R_1R_3 + R_1R_2 + R_2R_3)} \quad I_4 = \frac{\varepsilon(R_2 + R_3)}{(R_1R_3 + R_1R_2 + R_2R_3)}$$

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