Section 1.1 # 1 - 4. (Graphs)

Section 1.1 # 5. Write down a differential equation of the form $dy/dt = ay + b$ whose solutions all approach $y = 2/3$ as $t \to \infty$.

The equation can be solved as follows.

$$\frac{dy/dt}{y + b/a} = a,$$

$$\frac{d}{dt}\log(|y + b/a|) = a,$$
$$\log(|y + b/a|) = at + k,$$
$$y = ce^{at} - b/a.$$

To get solutions approaching $y = 2/3$, take $b/a = -2/3$ and $a = -3$ (any negative a works). Thus one possible equation is

$$dy/dt = -3y + 2.$$

Section 1.1 # 6. Write down a differential equation of the form $dy/dt = ay + b$ whose other solutions all diverge from $y = 2$ as $t \to \infty$.

The general solution method is the same as for problem 5.

To get solutions diverging from $y = 2$, take $b/a = -2$ and $a = 1$ (any positive a works). Thus one possible equation is

$$dy/dt = y - 2.$$

Section 1.1 # 18. A spherical drop evaporates at a rate proportional to its surface area. Write an equation for the volume.

The surface area A of a sphere of radius r is $4\pi r^2$, while the volume V is $\frac{4}{3}\pi r^3$. Thus

$$A = \frac{4\pi}{(4\pi/3)^{2/3}} V^{2/3} = (36\pi)^{1/3} V^{2/3}.$$

(Actually, the constant factor is not that important here.)

The equation is

$$\frac{dV}{dt} = CV^{2/3}.$$

The numerical constant relating A and V can be absorbed into C.

Section 1.1 # 19. Write an equation using Newton's law of cooling.

Let $T(t)$ be the object temperature, and T_s be the surrounding temperature. The general form is

$$\frac{dT}{dt} = r(T_s - T).$$

With the given numerical values we get

$$\frac{dT}{dt} = 0.05(70 - T).$$

There is a sign issue to worry about here. If T starts above T_s, then dT/dt should be negative. That is, the temperature of the object should be decreasing.

Section 1.1 # 20. A fluid containing $5mg/cm^3$ *is infused at the rate of* $100cm^3/h$. *The drug leaves the blood at a rate proportional to the amount present, with a rate constant of* $0.4/h$. *Find the equation for the amount of drug in the blood, and the long time behavior.*

Let $Q(t)$ be the mass of drug in the blood. The rate of change is the rate in minus the rate out, or

$$\frac{dQ}{dt} = 500mg/h - (0.4/h)Q.$$

Rewrite the equation as

$$\frac{dQ}{dt} = -0.4(Q - 1250).$$

Solve as above to get

$$Q(t) = Ce^{-0.4t} + 1250.$$

As $t \to \infty$ the amount of drug in the blood approaches $1250mg$.