Math 340 Professor Carlson
Homework 7

Section 3.3

#1 Euler' formula gives

$$\exp(2-3i) = e^2 e^{3i} = e^2\cos(3) + ie^2\sin(3).$$

#2 Euler' formula gives

$$e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1.$$

#5 To find the general solution of

$$y'' - 2y' + 2y = 0$$

find the roots of the characteristic equation $r^2 - 2r + 2 = 0$, which are $r = 1 \pm i$. The general solution is

$$y(t) = c_1 e^{(1+i)t} + c_2 e^{(1-i)t} = \alpha_1 e^t \cos(t) + \alpha_2 e^t \sin(t).$$

#6 To find the general solution of

$$y'' - 2y' + 6y = 0$$

find the roots of the characteristic equation $r^2 - 2r + 6 = 0$, which are $r = 1 \pm i\sqrt{5}$. The general solution is

$$y(t) = c_1 e^{(1+\sqrt{5}i)t)} + c_2 e^{(1+\sqrt{5}i)t)} = \alpha_1 e^t \cos(\sqrt{5}t) + \alpha_2 e^t \sin(\sqrt{5}t)$$

#7 To find the general solution of

$$y'' + 2y' + 2y = 0$$

find the roots of the characteristic equation $r^2 + 2r + 2 = 0$, which are $r = -1 \pm i$. The general solution is

$$y(t) = c_1 e^{(-1+i)t} + c_2 e^{(-1-i)t} = \alpha_1 e^{-t} \cos(t) + \alpha_2 e^{-t} \sin(t).$$

#8 To find the general solution of

$$y'' + 6y' + 13y = 0$$

find the roots of the characteristic equation $r^2 + 6r + 13 = 0$, which are $r = -3 \pm 2i$ The general solution is

$$y(t) = c_1 e^{(-3+2i)t} + c_2 e^{(-3-2i)t} = \alpha_1 e^{-3t} \cos(2t) + \alpha_2 e^{-3t} \sin(2t).$$

#12 *Solve the inital value problem*

$$y'' + 4y = 0, \quad y(0) = 0, y'(0) = 1.$$

The general solution is

$$y(t) = c_1 \cos(2t) + c_2 \sin(2t).$$

The initial conditions are satisfied when $c_1 = 0, \quad c_2 = 1/2$, so the desired solution is

$$y_1(t) = \frac{1}{2}\sin(2t).$$

#13 *Solve the inital value problem*

$$y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, y'(\pi/2) = 2.$$

The general solution is

$$y(t) = c_1 e^{(2+2i)t} + c_2 e^{(2-2i)t} = \alpha_1 e^{2t}\cos(2t) + \alpha_2 e^{2t}\sin(2t).$$

To find c_1, c_2 solve

$$y(\pi/2) = -\alpha_1 e^{\pi} = 0, \quad y'(\pi/2) = -2\alpha_1 e^{\pi} - 2\alpha_2 e^{\pi} = 2.$$

The initial conditions are satisfied when $\alpha_1 = 0, \quad \alpha_2 = -e^{-\pi}$, so the desired solution is

$$y_1(t) = -e^{-\pi} e^{2t}\sin(2t).$$

#21 Euler's formula gives

$$\frac{1}{2}[e^{it} + e^{-it}] = \frac{1}{2}[(\cos(t) + i\sin(t)) + (\cos(t) - i\sin(t))] = \cos(t),$$

$$\frac{1}{2i}[e^{it} - e^{-it}] = \frac{1}{2}[(\cos(t) + i\sin(t)) - (\cos(t) - i\sin(t))] = \sin(t).$$

Section 3.4

#1 To find the general solution of

$$y'' - 2y' + y = 0$$

find the roots of the characteristic equation $r^2 - 2r + 1 = 0$, which are $r = 1, 1$. The general solution is

$$y(t) = c_1 e^t + c_2 te^t.$$

#2 To find the general solution of

$$9y'' + 6y' + y = 0$$

find the roots of the characteristic equation $9r^2+6r+1=0$, which are $r=-1/3,-1/3$. The general solution is

$$y(t)=c_1e^{-t/3}+c_2te^{-t/3}.$$

#3 To find the general solution of

$$4y''-4y'-3y=0$$

find the roots of the characteristic equation $4r^2-4r-3=0$, which are $r=-1/2,3/2$ The general solution is

$$y(t)=c_1e^{-t/2}+c_2e^{3t/2}.$$

(This looks like a problem from the previous section.)

#9 *Solve the inital value problem*

$$9y''-12y'+4y=0, \quad y(0)=2, y'(0)=-1.$$

The roots of the characteristic equation $9r^2-12r+4=0$ are $r=2/3,2/3$, so the general solution is

$$y(t)=c_1e^{2t/3}+c_2te^{2t/3}.$$

The initial conditions give

$$y(0)=c_1=2, \quad y'(0)=(2/3)c_1+c_2=-1,$$

so the desired solution is

$$y_1(t)=2e^{2t/3}-te^{2t/3}.$$

#10 *Solve the inital value problem*

$$y''-6y'+9y=0, \quad y(0)=0, y'(0)=2.$$

The roots of the characteristic equation $r^2-6r+9=0$ are $r=3,3$, so the general solution is

$$y(t)=c_1e^{3t}+c_2te^{3t}.$$

The initial conditions give

$$y(0)=c_1=0, \quad y'(0)=3c_1+c_2=2,$$

so the desired solution is

$$y_1(t)=2te^{3t}.$$