Homework 5

1. By considering $B = -A$, show that a real symmetric matrix $\mathbf{A}$ is negative definite if and only $(-1)^k \Delta_k > 0$ for $k = 1, \ldots, N$.

2. Show that if a vector X of norm 1 maximizes the quadratic form

$$X \bullet \mathbf{A}X,$$

for a real symmetric matrix $\mathbf{A}$, then this vector must be an eigenvector corresponding to the largest eigenvalue of $\mathbf{A}$.

Hint: Write X as a linear combination of orthonormal eigenvectors, and see what happens when you compute $X \bullet \mathbf{A}X$.

3. Prove the following theorem. Hint: Write X as a linear combination of eigenvectors, and see what happens when you multiply by $\mathbf{A}^n$.

Theorem: *Suppose that $\mathbf{A}$ is a real symmetric positive definite matrix. If the vector X is not orthogonal to the eigenspace corresponding to the largest eigenvalue λ_1, then the sequence*

$$X_n = \frac{\mathbf{A}^n X}{\|\mathbf{A}^n X\|}$$

converges to an eigenvector of $\mathbf{A}$ with eigenvalue λ_1.

4. Find an orthonormal basis of eigenvectors for

$$\mathbf{A} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}.$$

Let $\mathbf{P}$ be the matrix whose columns are the orthonormal eigenvectors. Find the diagonal matrix $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ and use this to compute $\mathbf{A}^{10}$.

5. What happens in problem 3 if $\mathbf{A}$ is real symmetric, but not positive semidefinite?