Purpose of the experiment

- To measure a value of $g$, the acceleration of gravity at the Earth’s surface.
- To understand the relationships between position, velocity and acceleration.
- Use a Smartcart® to measure the speed and acceleration of a cart rolling down an incline.
- Determine the mathematical relationship between the angle of an incline and the acceleration of the cart down the ramp and using this relationship to find “$g$”.
- To help you see the importance of choosing the right tool when making a graph.

FYI

FYI If NASA sent birds into space they would soon die, they need gravity to swallow.
Kinematics

*Kinematics* is the branch of physics that describes motion. It answers questions concerning the motions of objects such as:

- “Where is the object?”
- “How fast is it going?”
- “In which direction is it moving?”
- “Is it speeding up, slowing down, or changing direction?”.

Bear in mind that while kinematics answers the question “How do objects move?”, it does not answer the question “Why do objects move?”. Answering “Why?” is left to the branch of physics called *dynamics*, which is the subject of a later lab.

Kinematics answers these questions with three quantities: position, velocity, and acceleration. Position tells us where the object is. Velocity tells us how fast it is going and in what direction. Acceleration tells us whether the object is speeding up, slowing down or changing direction.

In this lab, for simplicity’s sake, we will restrict ourselves to the idea of motion along a straight line, also referred to as one-dimensional motion. It is important to first understand one-dimensional motion before adding additional dimensions. Once you understand one-dimensional motion, it is easy to figure out two- and three-dimensional motions since they are simply two (or three) independent, perpendicular one-dimensional motions. However, for the purposes of this lab, the object will only be able to go forward/backward or, alternatively, up/down, and we don’t have to worry about whether or not it is turning. Two- and three-dimensional motion will be dealt with in a later lab.
Consider the kinematic equations of motion for constant acceleration. If we were to plot these equations on a graph, they would have a particular geometric shape. By comparing the kinematic equations to the equations for a parabola and straight line, you can see this more clearly:

- **Position vs. Time Equation**: \( y = \frac{1}{2} a t^2 + v_0 t + y_o \)
- **Equation of a Parabola**: \( y = A t^2 + B t + C \)

Here you can see that since \( A = a/2, B = v_o \) and \( C = y_o \), then the position vs. time graph must be a parabola. Also notice that:

- **Velocity vs. Time Equation**: \( v = a t + v_o \)
- **Equation of a Straight Line**: \( y = m x + b \)

Again, if I make the identifications that the slope \( m = a \) and the intercept \( b = v_o \), you can tell that the velocity vs. time graph must be a straight line. Remember this idea, we will come back to it in many future labs.

**Relationships between Kinematic Quantities**

Additionally, there is an important connection between the graph of position vs. time and graph of velocity vs. time. This relationship is stated as follows:

The slope at any given point on a position vs. time graph is the value at the same point on a velocity vs. time graph.

A similar relationship exists between the velocity vs. time graph and the acceleration vs. time graph:

The slope at any given point on a velocity vs. time graph is the value at the same point on an acceleration vs. time graph.

The second statement above is easily proven if you recall that the slope is simply the rise over the run for a given graph:

\[
slope \text{ on } v \text{ vs. } t = \frac{rise}{run} = \frac{\Delta v}{\Delta t} = \frac{(at_2+v_0)-(at_1+v_0)}{t_2-t_1} = \frac{a(t_2-t_1)}{t_2-t_1} = a = \text{value on } a \text{ vs. } t
\]
Thus, using the first statement above, if one knows an object’s position as a function of time, it is a simple matter to compute the velocity by finding the slope. The process can be repeated with the velocity graph to find the acceleration.

Is it also possible to reverse these relationships? For example, if we know the acceleration, can we find the velocity, and then repeat the process to find the position as a function of time? The answer is yes, and to do so we need to consider the area under the curve. These reverse relationships between acceleration and velocity is summarized as:

The area under the curve for a given time interval on an acceleration vs. time graph is the change in value \((\text{delta-v})\) over the same time interval on a velocity vs. time graph.

Likewise, a nearly identical relationship exists between velocity and position:

The area under the curve for a given time interval on a velocity vs. time graph is the change in value \((\text{displacement})\) over the same time interval on a position vs. time graph.

In this lab, we will use these concepts to analyze the motion of a ball. When a ball is tossed straight upward, the ball slows down until it reaches the top of its path. The ball then speeds up on its way back down. A graph of its velocity vs. time would show these changes. Is there a mathematical pattern to the changes in velocity? What is the accompanying pattern to the position vs. time graph? What would the acceleration vs. time graph look like?
During the early part of the seventeenth century, Galileo experimentally examined the concept of acceleration. One of his goals was to learn more about freely falling objects. Unfortunately, his timing devices were not precise enough to allow him to study free fall directly. Therefore, he decided to limit the acceleration by using fluids, inclined planes, and pendulums. In this lab exercise, you will see how the acceleration of a rolling ball or cart depends on the ramp angle. Then, you will use your data to extrapolate to the acceleration on a vertical “ramp;” that is, the acceleration of a ball in free fall, “g”.

The most direct way to measure “g” is to simply drop an object and measure it’s properties. The object will travel too quickly for a human to get accurate measurements over shorter distances. To overcome this defect in human reflexes, we need to slow the experiment down. Instead of a straight drop, we have to make the object travel down an inclined track. This will slow down the object and make measurements easier. This is not a direct measurement of “g”, but the reason all objects move “down hill” is because of gravitational attraction. Look at the following picture of a ball traveling down a track.

The ball wants to go straight down with an acceleration equal to g. The track forces the ball to travel along its surface.

The ball would like to fall to the ground in a straight line with an acceleration equal to “g”. The track forces the ball to travel down along its surface and will move with an acceleration related to “g” and the angle of the track. Think about this; if the track was inclined to 90° the ball would drop straight down and the acceleration of the ball would be equal to “g”. If we take the example of the opposite extreme, the track is not inclined at all (0°) the ball would not move at all and the acceleration would be equal to zero. This shows us that the acceleration of the ball down the track depends on how steep we make the track. We want to be somewhere in-between these two extremes. Rather than measuring time, as Galileo did, you will use a Smartcart® to determine the acceleration down inclines of various small angles.
This section is intended for those who have a more advanced math background, if you do not understand it yet don’t worry about it! Do however remember the resulting equation.

Here is a mathematical description of the experimental setup:

\[ \sum F_x = ma \]
\[ mg \sin \theta = ma \]

Cancel the m’s and:

\[ a = g \sin \theta \]

The acceleration of the object down the track (a) depends on the angle of the track (\( \theta \)) and the value of \( g \). The result we want to remember is:

\[ a = g \sin \theta \]

Equation 1

We can make a quick check to see if this equation makes sense by using the two extremes described earlier. If the track is inclined to 90\(^\circ\) (straight drop down) then equation 1 is:

\[ a = g \sin 90^\circ \]
\[ a = g \] which is just what we expect!

If the track is set at an angle of 0\(^\circ\), the track is flat and the object should not move. Putting this value into equation 1 gives:

\[ a = g \sin 0^\circ \]
\[ a = g \times 0 = 0 \] which means the object does not move!

Equation 1 seems to work!

Remember that the goal of this experiment is to find a value for \( g \). We can easily measure the angle of a track with a protractor. To find \( g \) we need to measure the acceleration of an object down the track. Then you can use Equation 1 to calculate the value of \( g \).
Say that’s great! Now, all we need to do is measure the acceleration \( a \). How do we do that?! If we look at the equations of motion, we see:

\[
\begin{align*}
\mathbf{v} &= a \mathbf{t} + \mathbf{v}_0 \\
\mathbf{x} &= \frac{1}{2} a \mathbf{t}^2 + \mathbf{v}_0 \mathbf{t} + \mathbf{x}_0
\end{align*}
\]

The way to get the value for \( a \), according to the equations of motion, is to either measure the velocity at different times or to measure the position at different times. Well, we can do both with the Smartcart®. Looking at the equations the velocity seems to be much simpler. We can make it even simpler if we start the cart from rest \( (\mathbf{v}_0 = 0) \). Therefore, the equation of motion then becomes:

\[
\mathbf{v} = a \mathbf{t} + 0 \quad \text{Equation 2}
\]

Let’s take a look at the equation of a straight line and that of the equation of motion in Equation 2 and see if there are any similarities:

<table>
<thead>
<tr>
<th>Equation of motion</th>
<th>[ \mathbf{v} = a \mathbf{t} + \mathbf{v}_0 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation of a straight line</td>
<td>[ \mathbf{y} = m \mathbf{x} + b ]</td>
</tr>
</tbody>
</table>

Looking at the symbols of these equations you might become confused because I’m saying that \( \mathbf{v} = \mathbf{y} \) and \( \mathbf{t} = \mathbf{x} \). What does this mean? Remember what each of these equations and symbols represent. If it helps you “read” the equations:

\[
\begin{align*}
\text{velocity} &= \text{(acceleration)} \times \text{(time)} + 0 \\
\text{vertical axis value} &= \text{(slope)} \times \text{(horizontal axis value)} + \text{(y-intercept)}
\end{align*}
\]

Try not to get caught up in the symbols and equations, but try to think about what the symbols mean.

Now you should be clear as to what the equations represent. You should also know that the equation of motion could be displayed as a straight line. How does this help us? If you take another look at the two equations, you will notice that the slope of the line is equal to the acceleration. Therefore, if you were to plot the velocity verses the time you should get a straight line whose slope is the acceleration of the object!

We can use this information to get \( g \). Remember that: \( a = g \sin \theta \).

With this single set of values of “\( a \)” and \( \theta \) we can calculate a value for \( g \). However, when dealing with a calculated result from experimental data the old adage applies:

MORE IS BETTER!

Kinematics - 7
Any single measurement can contain a large error. If you can gather many identical measurements error can be reduced.

One of the goals of this lab is to explore the usefulness of graphs so we are going to use a graph to find \( g \). Take another look at equation 1 and the equation of a straight line.

**Equation 1**  
Equation of a straight line

\[
\begin{align*}
    a &= g \sin \theta + 0 \\
    y &= m x + b
\end{align*}
\]

- acceleration (\( a \)) \iff vertical axis (\( y \))
- acceleration of gravity (\( g \)) \iff slope of the line (\( m \))
- \( \sin \) (incline angle, \( \theta \)) \iff horizontal axis (\( x \))
- starting position (\( x_0 = 0 \)) \iff y-intercept (\( b \))

From a graph of the acceleration verse the sine of the angle of the track (\( \sin \theta \)), we can finally get an experimental value for \( g \). The slope of the line is the value of \( g \).

Drawing a line lets you check the consistency of your data. A graph lets you check to see how close you are to the theoretical model, in this case a straight line.